1. [20pts] True (mark 0 ) or false (mark x) questions:
   (a) If A and B are regular, then so is A – B.
   (b) A(B∩C) = AB ∩ AC
   (c) A⊆ B implies A*⊆ B*
   (d) If A is regular, then \{ xx | x∈ A\} is regular.
   (e) If A is regular, then \{ x | xx∈ A\} is regular.
   (f) Every finite language is regular.
   (g) ((A*)*) * = A*
   (h) { ε } ∪ A^2 ⊆ A implies A = A*.
   (i) a(ba)* = (ab)*a
   (j) There is a set that is accepted by a NFA but could not be accepted by any DFA.

2. (6 pts) Let \Σ = \{ a, b\}. A = \{ ab, bc, ca\}, B=\{ ac, cb\}.
   (a) AB = ________________;
   (b) \Σ^3 - A = ________________.

3. (35 pts) Given the following NFA machine M :

   ![NFA Diagram]

   (a) Give a string of length 5 beginning with a and ending with b that is not accepted by M. [ 5pts]

   (b) Fill the following state transition table for the NFA M. Assume the states from left to right shown in the diagram are p, q, r and s, respectively. [5 pts]

<table>
<thead>
<tr>
<th>M</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(c) Let $M = (Q, \Sigma, \delta, S, F)$. Then what are the contents of $Q$, $\Sigma$, $\delta$, $S$ and $F$, respectively.

[5 pts]

$Q =$ _____________________________  $\Sigma =$ _____________________________

$S = \{p\}$  $F =$__________________________

$\delta =$

(d) List all the strings of length 3 accepted by $M$. [5 pts]

ANS: _______________________________________

(e) Find a regular expression equivalent to $M$. [5pts]

(f) Convert the automata $M$ to a DFA $M'$ by filling in the following table: [10pts]

<table>
<thead>
<tr>
<th>$M'$</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow{p}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“

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4. [15 pts] Consider the following regular set R represented by the regular expression:
\[ a^*b^* + b^*a^*. \]
(a) Draw a NFA for R. [5 pts]

(b) Draw a minimized DFA for R. [10 pts]

5. [15 pts] Let A be an infinite subset of the set \( \{ a^k b^k \mid k > 0 \} \). Using (game-theoretic) pumping lemma to show that A is not regular.
6. [14pts] Consider the following CFG:
   $S \rightarrow \text{ABS} \mid \text{AB} \quad A \rightarrow Aa \mid a \quad B \rightarrow bA$
   (a) Find a regular expression equivalent to the grammar. [6pts]
   (b) Find a derivation for the input ‘abaaba’ [4 pts]
   (c) Find all strings in {aabaab, aaaaab, aabbaa, ababab} that could be generated by G. [4 pts]

7. [15pts] Let $\Sigma$ be a finite alphabet. For any set $A \subseteq \Sigma^*$, $a \in \Sigma$, define
   $\text{permute}(A) = \{ y \mid \exists x \in A \text{ s.t. } y \text{ is a permutation of } x \}$,
   $\text{project}(A, a) = \{ a^p \mid \exists x \in A \text{ s.t. } p \text{ is the number of } a \text{'s occurring in } x \}.$

   For instance, if $A = \{aab, ab\}$, then $\text{permute}(A) = \{ab, ba, aab, aba, baa\}$ and $\text{project}(A, a) = \{aa, a\}$.
   (a) Show that if $A$ is regular, then so is $\text{project}(A, a)$. Hint: How to get $\text{project}(A, a)$ by
       applying a homomorphism to $A$? [7 pts]
   (b) Show that if $A$ is regular, then so is $\text{permute}(A)$. [8 pts]
A context-free grammar is said to be bilinear if every production is in one of the following forms: \( A \rightarrow aB, A \rightarrow Ba, A \rightarrow B, A \rightarrow \epsilon \), where \( A \) and \( B \) are non-terminals and \( a, b \) are any terminals. Bilinear grammar can accept non-regular sets and hence is more expressive than linear grammar. Now you are asked to design a bilinear grammar containing no more than 3 nonterminals that accepts the non-regular set \( L_1 = \{ a^n b^m | 0 \leq n \leq m \} \).